Functional Renormalization Group Method for BEC of Composite Particles

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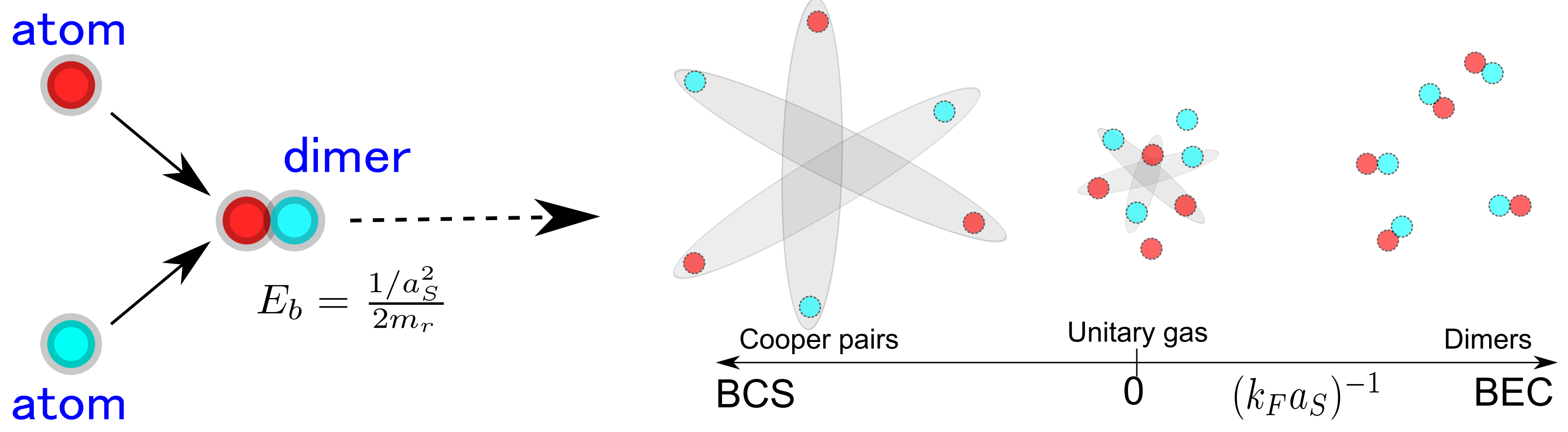
1. INTRODUCTION

- In the past decade strongly coupled fermionic superfluidity has been studied experimentally via cold atomic experiments, due to its importance in astro- and condensed matter physics.
- With a strong interaction, low-energy physics cannot be described by naive elementary excitations. Pseudo-gap appears and composite-particle excitations can play an important role.
- In order to study strongly-correlated systems systematically, we developed a new formalism of the Functional Renormalization Group (FRG).

Model: two-component fermionic systems with a contact interaction:

$$S=\int_0^\beta \mathrm{d}\tau \int \mathrm{d}^3\boldsymbol{x}\left[\overline{\psi}\left(\partial_\tau-\frac{\nabla^2}{2m}-\mu\right)\psi+g\overline{\psi}_\uparrow\overline{\psi}_\downarrow\psi_\downarrow\psi_\uparrow\right]$$

(β^{-1}: temperature, m: mass, μ: chemical potential, g: bare attractive coupling)



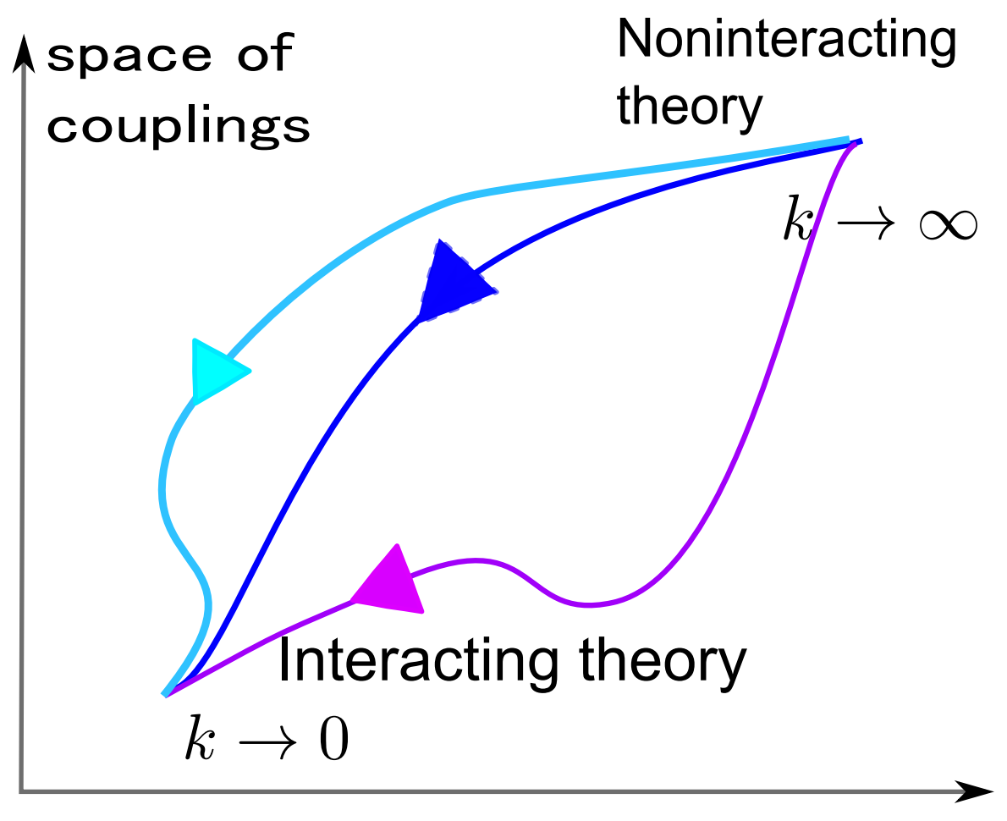
Goals:

- Describe the Bose-Einstein condensation of dimers from the viewpoint of the field-theoretic RG approach in the language of original fermionic degrees of freedom.
- Develop fermionic FRG in a systematic, improvable way without the help of auxiliary bosonic fields.

2. FUNCTIONAL RENORMALIZATION GROUP

Why fermionic FRG?

- Unbiased and nonperturbative study of interacting fermion systems.
- More natural to apply improvable approximations.
- Scale separation enables to observe the physical origin of gradual corrections.



Basic strategy: FRG introduces a new term $\delta S_k[\overline{\psi},\psi]$, which regulate infrared (IR) behaviors of interacting systems (IR regulator).
Definition of the Schwinger functional $W_k[J]$:

$$\exp(W_k[J])=\int\mathcal{D}\phi\ \exp\left[-(S[\phi]+\delta S_k[\phi])+J\cdot\phi\right]$$

- $k\to\infty$: Noninteracting theory. It is very easy to calculate $W[J]$!
- $k\to 0$: Original interacting theory. Calculation of $W[J]$ is the goal of Many-Body Physics.
- Interacting theory ($k=0$) can be studied by gradually changing the parameter k starting from the noninteracting theory ($k=\infty$):

$$-\partial_k W_k[J]=\exp(-W_k[J])\left(\partial_k\delta S_k\left[\frac{\delta_L}{\delta J}\right]\right)\exp(W_k[J]).$$

Optimization of the RG flow: Choice of the IR regulator δS_k is arbitrary.
The optimization criterion provides an indication of the controllability of the FRG equation [1, 2]:

- IR regulators δS_k make the system gapped by a typical energy $k^2/2m$ of the parameter k.
- High-energy excitations with excitation energy $\gtrsim k^2/2m$ must decouple from the RG flow.

3. LOW-ENERGY DIMERS AND VERTEX IR REGULATOR

When a_s is positive, the system consists of weakly-interacting dimers.
Four point function in the vacuum:

$$\Gamma^{(4)}(p)\simeq\frac{-(8\pi/m^2a_s)}{ip^0+\boldsymbol{p}^2/4m}$$

with $\mu=-1/2ma_s^2$. There exists a low-energy excitation as a pole of a four-point vertex.

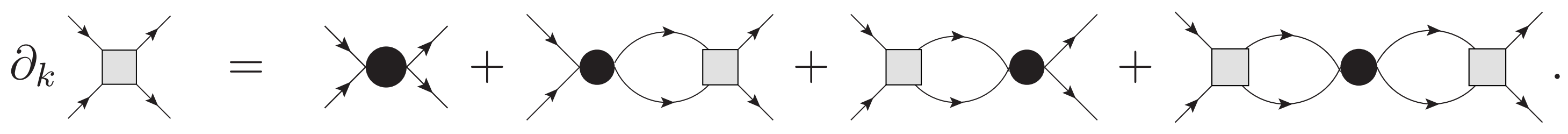
⇒ Optimization can be satisfied with the vertex IR regulator [3]:

$$\delta S_k[\overline{\psi},\psi]=\int_p\frac{g^2R_k(\boldsymbol{p})}{1-gR_k(\boldsymbol{p})}\int_{q,q'}\overline{\psi}_{\uparrow,p/2+q}\overline{\psi}_{\downarrow,p/2-q}\psi_{\downarrow,p/2-q'}\psi_{\uparrow,p/2+q'}.$$

4. FRG FLOW WITH THE VERTEX IR REGULATOR

Flow of the four-point vertex:
In the low-density limit, the flow equation of the four-point vertex is given by

$$\begin{aligned}\partial_k\Gamma_k^{(4)}(p;q,q') &= \partial_k g_k(p)-\int_l\left(\frac{(\partial_k g_k)(p)\Gamma_k^{(4)}(p;l,q')}{G^{-1}(\frac{p}{2}+l)G^{-1}(\frac{p}{2}-l)}+\frac{\Gamma_k^{(4)}(p;q,l)(\partial_k g_k)(p)}{G^{-1}(\frac{p}{2}+l)G^{-1}(\frac{p}{2}-l)}\right)\\ &\quad+\int_{l,l'}\frac{\Gamma_k^{(4)}(p;q,l)(\partial_k g_k)(p)\Gamma_k^{(4)}(p;l',q')}{G^{-1}(\frac{p}{2}+l)G^{-1}(\frac{p}{2}-l)G^{-1}(\frac{p}{2}+l')G^{-1}(\frac{p}{2}-l')},\end{aligned}$$

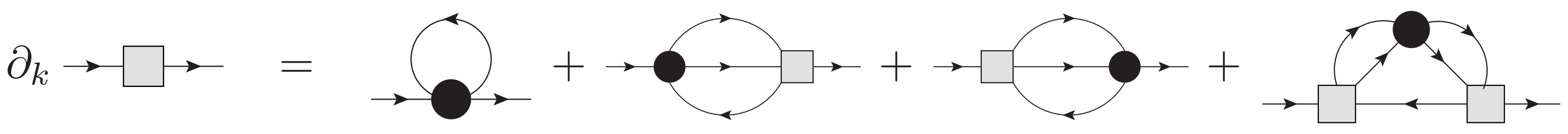


This approximation is equivalent to the PP-RPA:

$$\frac{1}{\Gamma_k^{(4)}(p)}=-\frac{m^2a_s}{8\pi}\left(ip^0+\frac{\boldsymbol{p}^2}{4m}\right)-R_k(\boldsymbol{p}).$$

⇒ Low-energy dimer excitations are suppressed thanks to the vertex IR regulator!
Optimization can be satisfied.

Flow of the self-energy: Diagrammatic expression of the self-energy flow equation is



Using the PP-RPA of the four-point vertex $\Gamma_k^{(4)}$, the flow equation becomes

$$\partial_k\Sigma_k(p)=\int_l^{(T)}\frac{\partial_k\Gamma_k^{(4)}(p+l)}{G^{-1}(l)-\Sigma_k(l)}.$$

5. SELF-ENERGY FLOW AND FREE BEC

If the self-energy is small, we can easily solve the flow equation of Σ.
As long as $|\Sigma(p)|\ll|\mu|(=1/2ma_s^2)$, the self-energy in the fermion propagator is negligible:

$$\begin{aligned}\Sigma_k(p) &= \int_l^{(T)}\frac{\Gamma_k^{(4)}(p+l)}{il^0+\boldsymbol{l}^2/2m+1/2ma_s^2}\\ &\simeq \int\frac{\mathrm{d}^3\boldsymbol{q}}{(2\pi)^3}\frac{(8\pi/m^2a_s)n_B\left(\boldsymbol{q}^2/4m+\widetilde{R}_k(\boldsymbol{q})\right)}{ip^0+\boldsymbol{q}^2/4m+\widetilde{R}_k(\boldsymbol{q})-(\boldsymbol{q}+\boldsymbol{p})^2/2m-1/2ma_s^2}.\end{aligned}$$

with n_B the Bose-Einstein distribution function, and $\widetilde{R}_k=\frac{8\pi}{m^2a_S}R_k$.

- This self-energy has a nontrivial momentum dependence.
- The optimization controls the RG flow and validates a simple approximation.

Estimate of the magnitude of the self-energy:
Even for small k compared with the scale of temperature, the magnitude of the self-energy can be estimated as

$$|\Sigma_k(p)|\lesssim\frac{1}{2ma_s^2}\times(\sqrt{2mT}a_s)^3\times n_B(k^2/4m).$$

This quantity can be much smaller than the binding energy. The critical value of k is

$$\frac{k_c^2/2m}{T}\sim(\sqrt{2mT}a_s)^3(\ll 1).$$

Number density and free BEC:
With the fermion self-energy, the number density can be evaluated as

$$n=\int_p\frac{1}{ip^0+\boldsymbol{p}^2/2m+1/2ma_s^2-\Sigma_0(p)}=\frac{(2mT_c)^{3/2}}{\pi^2}\sqrt{\frac{\pi}{2}}\zeta(3/2).$$

This is nothing but the transition temperature of the BEC of the free Bose gas.

6. CONCLUSIONS

- We proposed a new formalism of FRG introducing the vertex IR regulator. Usefulness of the vertex IR regulator is shown when low-energy physics is described by composite particles.
- When the effective theory contains not only low-energy degrees of freedom but also high-energy ones, the nonlocal property of Green functions plays an important role.
- This study shows that the optimization criterion provides an important guiding principle to attack strongly-interacting systems with FRG.

References

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